PHYSICAL REVIEW E VOLUME 60, NUMBER 2 AUGUST 1999

Universal critical coupling constants for the three-dimensional *n*-vector model from field theory

A. I. Sokolov,[1,2] E. V. Orlov,[1] V. A. Ul'kov,[2] and S. S. Kashtanov[1]
[1]*Department of Physical Electronics, Saint Petersburg Electrotechnical University, Professor Popov Street 5, St. Petersburg 197376, Russia*
[2]*Department of Physics, Saint Petersburg Electrotechnical University, Professor Popov Street 5, St. Petersburg 197376, Russia*
(Received 29 March 1999)

The field-theoretical renormalization group (RG) approach in three dimensions is used to estimate the universal critical values of renormalized coupling constants g_6 and g_8 for the $O(n)$-symmetric model. The RG series for g_6 and g_8 are calculated in the four- and three-loop approximations, respectively, and then resummed by means of the Padé-Borel-Leroy technique. Under the optimal value of the shift parameter b providing the fastest convergence of the iteration procedure, numerical estimates for g_6^* are obtained with an accuracy no worse than 0.3%. The RG expansion for g_8 demonstrates a stronger divergence, and results in considerably cruder numerical estimates. [S1063-651X(99)01808-5]

PACS number(s): 64.60.Fr, 05.70.Jk, 11.10.Gh, 64.60.Ak

I. INTRODUCTION

The three-dimensional (3D) $O(n)$-symmetric model plays a very important role in the theory of phase transitions. It describes critical phenomena in a variety of physical systems including Ising, XY-like, and Heisenberg ferromagnets, simple fluids and binary mixtures, superconductors and Bose superfluids, etc. This model is also relevant to certain asymptotic regimes of the critical behavior of the quark-gluon plasma in quantum chromodynamics ($n=4$) [1,2]. In the critical region, the *n*-vector model is known to be thermodynamically equivalent to the 3D Euclidean field theory of $\lambda\varphi^4$ type, and may be treated by the field-theoretical renormalization group (RG) technique which proved to be very efficient both for studying the qualitative features of phase transitions and calculating the critical exponents [3–7].

On the other hand, for decades the influence of ordering fields upon the critical behavior of various systems attracted permanent attention, being of prime interest both for theorists and experimentalists. Recently, the free energy (effective action) and, in particular, higher-order renormalized coupling constants g_{2k} for the basic models of phase transitions became the target of intensive theoretical studies [7–23]. These constants are related to the non-linear susceptibilities χ_{2k} and enter the scaling equation of state, thus playing a key role at criticality. Along with critical exponents and critical amplitude ratios, they are universal, i.e., they possess, under $T \to T_c$, numerical values that are not sensitive to the physical nature of the phase transition, depending only on the system dimensionality and the symmetry of the order parameter.

Calculation of the universal critical values of g_6, g_8, etc. for the three-dimensional Ising model by a number of analytical and numerical methods showed that the field-theoretical RG approach in fixed dimensions yields the most accurate numerical estimates for these quantities. It is a consequence of a rapid convergence of the iteration schemes originating from renormalized perturbation theory. Indeed, the resummation of four- and five-loop RG expansions by means of the Borel-transformation-based procedures gave values for g_6^* which differ from each other by less than 0.5% [18,19], while the use of a resummed three-loop RG expansion enabled one to achieve an apparent accuracy no worse than 1.6% [7,17]. Moreover, the field-theoretical RG approach turns out to be powerful enough even in two dimensions: properly resummed four-loop RG expansions lead to fair numerical estimates for the critical exponents [3] and the renormalized coupling constant g_6^* [24] of a 2D Ising model, and give reasonable results for its random counterpart [25]. It is natural, therefore, to use the field theory for a calculation of renormalized higher-order coupling constants for the 3D *n*-vector model. In this paper, the 3D RG expansion for the renormalized coupling constants g_6 and g_8 will be calculated, and the numerical estimates for their universal critical values will be obtained.

II. RG EXPANSIONS FOR THE SEXTIC AND OCTIC COUPLING CONSTANTS

Within field-theoretical language, the 3D $O(\mathrm{n})$-symmetric model in the critical region is described by Euclidean scalar field theory with the Hamiltonian

$$H=\int d^3x\left[\tfrac{1}{2}(m_0^2\varphi_\alpha^2+(\nabla\varphi_\alpha)^2)+\lambda(\varphi_\alpha^2)^2\right], \tag{1}$$

where a bare mass squared m_0^2 is proportional to $T-T_c^{(0)}$, $T_c^{(0)}$ being the phase transition temperature in the absence of the order parameter fluctuations. Taking fluctuations into account results in renormalizations of the mass $m_0 \to m$, the field $\varphi \to \varphi_R$, and the coupling constant $\lambda \to m g_4$. Moreover, thermal fluctuations give rise to many-point correlations $\langle\varphi(x_1)\varphi(x_2)\cdots\varphi(x_{2k})\rangle$ and, correspondingly, to higher-order terms in the expansion of the free energy in powers of the magnetization M:

$$F(M,m)=F(0,m)+\sum_{k=1}^{\infty}\Gamma_{2k}M^{2k}. \tag{2}$$

In the critical region, the coefficients Γ_{2k}, being one-particle irreducible $2k$-point vertices taken at zero external momenta, demonstrate the well-known scaling behavior

$$\Gamma_{2k}=g_{2k}m^{3-k(1+\eta)}, \tag{3}$$

where η is a Fisher exponent, and g_{2k} are some constants. Let us set as usual, $g_2=\frac{1}{2}$. Then g_4, g_6, g_8, . . . will acquire universal values. The asymptotic critical values of g_4, $g_4^*(n)$, determine the critical exponents and other universal quantitites, thus playing a very important role in the theory. The numbers $g_4^*(n)$ have been found by resummation of the six-loop expansion for the RG β function [3,4,6,7], from strong-coupling series [26], by lattice calculations [21], and from the ϵ expansion [27], and are known today with an accuracy which may be considered rather high.

The universal values of higher coupling constants g_6^*, g_8^*, etc. determine the structure of the free energy $F(M,m)$ under strong critical fluctuations. In fact, the Taylor expansion of the scaling function contains the ratios $g_{2k}^*/(g_4^*)^{k-1}$, which may be easily shown by replacement of the magnetization M in Eq. (2) by the dimensionless variable $z=M\sqrt{g_4/m^{1+\eta}}$:

$$F(z,m)-F(0,m)=\frac{m^3}{g_4}\left(\frac{z^2}{2}+z^4+\frac{g_6}{g_4^2}z^6+\frac{g_8}{g_4^3}z^8+\cdots\right). \tag{4}$$

Moreover, via g_{2k}, the nonlinear susceptibilities χ_{2k} can be expressed. For χ_4 and χ_6, corresponding formulas are as follows:

$$\chi_4=\left.\frac{\partial^3 M}{\partial H^3}\right|_{H=0}=-24\chi_2^2 m^{-3}g_4,$$

$$\chi_6=\left.\frac{\partial^5 M}{\partial H^5}\right|_{H=0}=720\chi_2^3 m^{-6}(8g_4^2-g_6). \tag{5}$$

Their inversion gives the relations

$$g_4=-\frac{m^3\chi_4}{24\chi_2^2},\quad g_6=\frac{m^6(10\chi_4^2-\chi_6\chi_2)}{720\chi_2^4}, \tag{6}$$

which are widely used for extraction of numerical values of renormalized coupling constants from the results of lattice calculations [14,16,21,22,28,29].

The method of calculating the RG series for the g_6 and g_8 we use here is straightforward. Since in three dimensions higher-order bare couplings are irrelevant in the RG sense, the renormalized perturbative series to be found can be obtained from conventional Feynman graph expansions for the six- and eight-point vertices in terms of the only bare coupling constant $-\lambda$. In the course of calculations the tensor structures of these vertices,

$$\Gamma_{\alpha\beta\gamma\delta\mu\nu}=\tfrac{1}{15}(\delta_{\alpha\beta}\delta_{\gamma\delta}\delta_{\mu\nu}+14\ \text{transpositions})\Gamma_6 \tag{7}$$

$$\Gamma_{\alpha\beta\gamma\delta\mu\nu\rho\sigma}=\tfrac{1}{105}(\delta_{\alpha\beta}\delta_{\gamma\delta}\delta_{\mu\nu}\delta_{\rho\sigma}+104\ \text{transpositions})\Gamma_8, \tag{8}$$

should be taken into account. In its turn, λ may be expressed perturbatively as a function of the renormalized coupling constant g_4. Substituting corresponding power series for λ into original expansions, we can obtain the RG series for g_6 and g_8. The one-, two-, three-, and four-loop contributions to g_6 are formed by one, three, 16, and 94 one-particle irreducible Feynman graphs, respectively. Their calculation gives

$$g_6=\frac{9}{\pi}\left(\frac{\lambda Z^2}{m}\right)^3\left[\frac{n+26}{27}-\frac{9n^2+340n+2324}{162\pi}\left(\frac{\lambda Z^2}{m}\right)+(0.0056289546468n^3+0.28932672886n^2+4.0404241235n+16.204286853)\left(\frac{\lambda Z^2}{m}\right)^2-(0.001493126n^4+0.09961447n^3+2.152320n^2+18.330704n+52.830284)\left(\frac{\lambda Z^2}{m}\right)^3\right]. \tag{9}$$

The perturbative expansion for λ emerges directly from the normalizing condition $\lambda=mZ_4Z^{-2}g_4$ and the known series for Z_4 [6]:

$$Z_4=1+\frac{n+8}{2\pi}g_4+\frac{3n^2+38n+148}{12\pi^2}g_4^2+(0.0040314418n^3+0.0679416657n^2+0.466356233n+1.240338484)g_4^3. \tag{10}$$

Combining these expressions, we obtain

$$g_6=\frac{9}{\pi}g_4^3\left[\frac{n+26}{27}-\frac{17n+226}{81\pi}g_4+(0.000999164n^2+0.14768927n+1.24127452)g_4^2-(-0.00000949n^3+0.00783129n^2+0.34565683n+2.14825455)g_4^3\right]. \tag{11}$$

In the case of g_8, the one-, two-, and three-loop contributions are given by one, five, and 36 Feynman graphs, respectively. Corresponding ‘‘bare’’ and renormalized perturbative expansions are found to be

TABLE I. The values of g_6^* for $n=1$, 3, and 10 obtained by means of the Padé-Borel-Leroy technique for various b within three-loop (approximant [1/1]) and four-loop (approximants [1/2] and [2/1]) RG approximations. The estimates for several values of b in the middle lines are absent because corresponding Padé approximant turns out to be spoiled by a positive axis pole.

b	0	1	1.24	2	3	4	5	7
$n=1$								
[1/1]	1.576	1.604	1.6089	1.621	1.633	1.641	1.648	1.656
[1/2]	-	-	1.6084	1.600	1.595	1.592	1.590	1.587
[2/1]	1.639	1.613	1.6084	1.596	1.583	1.573	1.566	1.555
$n=3$								
[1/1]	0.937	0.949	0.95133	0.957	0.962	0.966	0.969	0.973
[1/2]	-	-	0.95133	0.948	0.946	0.944	0.944	0.942
[2/1]	0.964	0.953	0.95133	0.946	0.941	0.937	0.934	0.930
$n=10$								
[1/1]	0.2338	0.23515		0.2360	0.2366	0.2370	0.2373	0.2377
[1/2]	-	-		0.2348	0.2346	0.2345	0.2344	0.2342
[2/1]	0.2359	0.23515		0.2346	0.2342	0.2339	0.2337	0.2334

$$g_8=-\frac{81}{2\pi}\left(\frac{\lambda Z^2}{m}\right)^4\left[\frac{n+80}{81}-\frac{405n^2+35626n+342320}{13122\pi}\left(\frac{\lambda Z^2}{m}\right)+(0.0046907955n^3+0.463650683n^2+8.86811653n+45.4769028)\left(\frac{\lambda Z^2}{m}\right)^2\right], \tag{12}$$

$$g_8=-\frac{81}{2\pi}g_4^4\left[\frac{n+80}{81}-\frac{81n^2+7114n+134960}{13122\pi}g_4+(0.00943497n^2+0.60941312n+7.15615323)g_4^2\right]. \tag{13}$$

In Sec. III, the series of equations (11) and (13) will be used for estimation of the universal numbers g_6^* and g_8^*.

III. RESUMMATION AND NUMERICAL ESTIMATES

Being a field-theoretical perturbative expansions the series of equations (11) and (13) have factorially growing coefficients, i.e., they are divergent (asymptotic). Hence, direct substitution of the fixed point value g_4^* into them would not lead to satisfactory results. To obtain reasonable numerical estimates for g_6^* and g_8^*, some procedure making these expansions convergent should be applied. As is well known, the Borel-Leroy transformations

$$f(x)=\sum_{i=0}^{\infty}c_ix^i=\int_0^{\infty}t^be^{-t}F(xt)dt,\quad F(y)=\sum_{i=0}^{\infty}\frac{c_i}{(i+b)!}y^i, \tag{14}$$

diminishing the coefficients by the factor $(i+b)!$, can play a role of such a procedure. Since the RG series considered turns out to be alternating the analytical continuation of the Borel transform may be then performed by using Padé approximants.

Let us discuss first the estimation of the sextic coupling constant g_6^*. With the four-loop expansion (11) in hand, we can construct, in principle, three different Padé approximants: [2/1], [1/2], and [0/3]. To obtain proper approximation schemes, however, only diagonal $[L/L]$ and near-diagonal Padé approximants should be employed [30]. That is why, further, when estimating g_6^*, we limit ourselves with approximants [2/1] and [1/2]. Moreover, the diagonal Padé approximant [1/1] will also be dealt with, although this corresponds, in fact, to the usage of the lower-order, three-loop RG approximation.

The algorithm of estimating g_6^* we use here is as follows. Since the Taylor expansion for the free energy contains as coefficients the ratios $R_{2k}=g_{2k}/g_4^{k-1}$ rather than the renormalized coupling constants themselves, we work with the RG series for R_6. It is resummed in three different ways based on the Borel-Leroy transformation and the Padé approximants just mentioned. The Borel-Leroy integral is evaluated as a function of the parameter b under $g_4=g_4^*$. For the fixed point coordinate g_4^*, the values given by the resummed six-loop RG expansion for the β function are adopted [3,7], which are believed to be the most accurate estimates available today. The optimal value of b providing the fastest convergence of the iteration scheme is then determined. It is deduced from the condition that the Padé approximants employed should give, for $b=b_{\mathrm{opt}}$, the values of R_6^* which are as close as possible to each other. Finally, the average over three estimates for R_6^* is found and claimed to be a numerical value of this universal ratio.

To obtain an idea about how such a procedure works, let us use Table I, where the results of corresponding calculations for $n=1$, 3, and 10 are presented. It is seen that for $n=1$ and 3, b_{opt}, providing a coincidence of the estimates given by all three working Padé approximants, is equal to 1.24. For $n=10$, b_{opt}, fixed by the approximants [1/1] and [2/1], is equal to 1, whereas the third approximant ([1/2]) at

TABLE II. Our estimates of universal critical values of the renormalized sextic coupling constant for the 3D n-vector model (column 3). The fixed point coordinates g^* are taken from Refs. [7] ($1\leq n\leq 3$) and [11] ($4\leq n\leq 40$). The g_6^* estimates extracted earlier from Pade-Borel resummed three-loop RG expansion (column 4), from the exact RG equations (column 5), obtained by the lattice calculations (column 6), and resulting from a constrained analysis of the ϵ-expansions (column 7) are presented for comparison. Column 8 contains the values of g_6^* given by the $1/n$-expansion technique.

n	g^*	g_6^*	g_6^* [7]	g_6^* [11]	g_6^* [14]	g_6^* [23]	g_6^* $(1/n)$
	2	3	4	5	6	7	8
1	1.415	1.608	1.622	1.52	1.92(24)	1.609(9)	
2	1.406	1.228	1.236	1.14	1.27(25)	1.21(7)	
3	1.392	0.951	0.956	0.88	0.93(20)	0.931(46)	
4	1.3745	0.747	0.751	0.68	0.62(15)	0.725(29)	1.6449
5	1.3565	0.596	0.599				1.0528
6	1.3385	0.483	0.485				0.7311
7	1.321	0.396	0.398				0.5371
8	1.3045	0.329	0.331			0.319(4)	0.4112
9	1.289	0.277	0.278				0.3249
10	1.2745	0.235	0.236				0.2632
12	1.2487	0.174	0.175				0.1828
14	1.2266	0.134	0.134				0.1343
16	1.2077	0.105	0.105			0.1032(4)	0.1028
18	1.1914	0.0845	0.0847				0.0812
20	1.1773	0.0693	0.0694				0.0658
24	1.1542	0.0487	0.0488				0.0457
28	1.1361	0.0360	0.0361				0.0336
32	1.1218	0.0276	0.0276			0.0275(1)	0.0257
36	1.1099	0.0218	0.0218				0.0203
40	1.1003	0.0176	0.0176				0.0164

$b=1$ is spoiled by a positive axis pole. Nevertheless, the numerical estimate given by this approximant under the nearest ''safe'' (integer) value of $b(b=2)$ turns out to be very close to that predicted by the pole free approximants for $b_{\rm opt}$. Moreover, as is seen from Table I, with increasing n numerical estimates for g_6^* become less dependent on b, i.e., their sensitivily to the type of resummation decreases. This is not surprising. The point is that the RG expansion (11) becomes less divergent when n grows up. To make this property obvious, let us replace g_4 in Eq. (11) by the effective coupling constant

$$g=\frac{n+8}{2\pi}g_4, \tag{15}$$

that is known to be only weakly dependent on n: it varies from 1.415 to 1 when n goes from 1 to infinity [6,7]. Then we obtain

$$g_6=\frac{8\pi^2(n+26)}{3(n+8)^3}g^3\Bigg[1-\frac{2(17n+226)}{3(n+8)(n+26)}g+\frac{1.065025n^2+157.42454n+1323.09596}{(n+8)^2(n+26)}g^2 - \frac{-0.0638n^3+52.4510n^2+2314.9897n+14387.6460}{(n+8)^3(n+26)}g^3\Bigg]. \tag{16}$$

One can see now that all terms in the RG expansion for g_6 (in square brackets), apart from the first one, decrease monotonically when $n\to\infty$. This implies that the larger n is the smaller the contribution of the higher-order terms and, correspondingly, the better the approximating properties of this series.

This conclusion is definitely confirmed by Table II. It contains numerical estimates for g_6^* resulting from the four-loop RG expansion resummed by the Padé-Borel-Leroy technique described above (column 3), and their analogs given by the Padé-Borel resummed three-loop RG series [7] (column 4). As is seen, with increasing n the difference between the four- and three-loop estimates rapidly diminishes. Being small (0.9 %) even for $n=1$, it becomes negligible at n

TABLE III. Three-loop RG estimates of universal critical values of the renormalized octic coupling constant g_8 (column 2). The g_8^* estimates resulting from a constrained analysis of the ϵ-expansion (column 3), from the exact RG equations (column 4) and given by the $1/n$-expansion technique (column 5) are presented for comparison.

n	g_8^*	g_8^* [23]	g_8^* [11]	g_8^* $(1/n)$
	2	3	4	5
1	0.825	0.82(9)	0.721	
2	0.388	0.83(31)	0.343	
3	0.168	0.36(17)	0.145	
4	0.057	0.15(13)	0.042	−2.151
6	−0.021			−0.834
8	−0.034	−0.03(2)		−0.0388
16	−0.014	−0.015(2)		−0.0456
32	−0.0023	−0.0023(1)		−0.00395
48	−0.00062	−0.00061(2)		−0.00087
64	−0.00023			−0.00029
100	−0.000046	−0.000044(2)	−0.000049	−0.000052

$=10$ and practically disappears for $n \geqslant 14$.

How close to the exact values of g_6^* may the numbers in column 3 be? To clear up this point, let us compare our four-loop estimate for R_6^* at $n=1$ with those obtained recently by an analysis of the five-loop scaling equation of state for the 3D Ising model [19,31]. Guida and Zinn-Justin obtained $R_6^*=1.644$ and, taking into account some additional information, $R_6^*=1.643$, while our estimate is $R_6^*=1.648$. Keeping in mind that the exact value of R_6^* should lie between the four- and five-loop estimates (the RG series is alternating), our estimate can differ from the exact number by no more than 0.3%. Since for $n>1$ the RG expansion (11) was argued to provide better numerical estimates than in the Ising case, this value (0.3%) may be referred to as an upper bound for the deviation of the numbers in column 3 of Table II, from their exact counterparts.

It is interesting to compare our estimates for g_6^* with those obtained by other methods. Since 1994, the universal values of the sextic coupling constant for the 3D $O(n)$-symmetric model were estimated by solving the exact RG equations [11], by lattice calculations [14], and by a constrained analysis of the ϵ-expansion [23]; corresponding results are collected in columns 5, 6, and 7 of Table II, respectively. As is seen, they are, in general, in accord with ours.

A less optimistic situation takes place in the case of the octic coupling constant g_8. The RG expansion [Eq. (13)] is shorter than Eq. (11), and strongly diverges. Moreover, the second term in this series, along with the first one, remains finite under $n\to\infty$. It becomes obvious if one replaces g_4 by g:

$$g_8=-\frac{8\pi^3(n+80)}{(n+8)^4}g^4\left[1-\frac{81n^2+7114n+134960}{81(n+80)(n+8)}g+\frac{30.1707n^2+1948.7519n+22883.6021}{(n+80)(n+8)^2}g^2\right]. \quad (17)$$

In addition, the RG series for g_8 has an unusual feature: when $n\to\infty$, the first and second terms tend to compensate for each other, making their mutual contribution small and increasing the role of the higher-order terms. That is why numerical estimates resulting from expansion (17) are expected to be substantially cruder than those given by series (16) both for small and large values of n.

In order to estimate $g_8^*(n)$, we resum the RG expansion for g_8 by the Padé-Borel-Leroy technique using the diagonal Padé approximant [1/1]. Other Padé approximants, [0/2] and [0/1], are ignored, since they turn out to lead to quite unsatisfactory numerical results. Dealing with a single Padé approximant, in some condition we need to fix the optimal value of the shift parameter b. For the three-dimensional Ising model the estimate $g_8^*=0.825$ was recently found [19]. This number has been extracted from the five-loop RG expansion, and may be considered the most accurate known up to the present. It is natural therefore to tune, by proper choice of b, a numerical value of $g_8^*(1)$ given by the resummed three-loop RG series with the best estimate available. Such a procedure leads to $b_{\rm opt}=40$, and this number is adopted as optimal in the course of evaluation of g_8^* for arbitrary n.

The results of our calculations are collected in Table III, where the estimates for $g_8^*(n)$, obtained by a constrained analysis of the ϵ expansion [23] by an approximate solution of the exact RG equations [11] and given by the $1/n$-expansion technique, are also presented for comparison. As seen, for $n \geqslant 8$ the numbers originating from two field-theoretical approaches—g expansion in three dimensions and ϵ expansion—agree quite well. However, for smaller n, especially for $n=2$, differences between them turn out to be rather large. This is not surprising since overly short perturbative expansions for g_8 are available both in three and $4-\epsilon$, dimensions and they demonstrate a strong divergence preventing accurate numerical estimates from being obtained. At the same time, our three-loop RG estimates are believed to be closer to the true critical values of g_8 than those given by the ϵ expansion, because in three dimensions we have longer perturbative series. A fair agreement between our results and the numbers emerging from the exact RG equations (see Table III) may be considered as an argument in favor of this belief.

IV. CONCLUSION

To summarize, we have calculated the RG expansions for renormalized coupling constants g_6 and g_8 of the 3D n-vector model in four- and three-loop orders, respectively. Resummation of the RG series by the Padé-Borel-Leroy method has enabled us to obtain numerical estimates for the universal critical values of these quantities for arbitrary n. Having analyzed the sensitivity of the RG estimates for g_6^* to the type of resummation procedure and a character of their dependence on the order of the RG approximation, the apparent accuracy of these numbers has been argued to be no worse than 0.3%. Numerical estimates for g_8^* turned out to be less accurate because of the smaller length and stronger divergence of the RG expansion obtained. They were found to be consistent, in general, with the values of g_8^* deduced from the exact RG equations and, for $n \geqslant 8$, with those given by a constrained analysis of corresponding ϵ expansion.

ACKNOWLEDGMENTS

This work was supported by the Ministry of General and Professional Education of the Russian Federation under Grant No. 97-14.2-16. One of the authors (A.I.S.) gratefully acknowledges the support of the International Science Foundation via Grant No. p98-537.

[1] F. Wilczek. Int. J. Mod. Phys. A **7**, 3911 (1992).
[2] K. Rajagopal and F. Wilczek, Nucl. Phys. B **399**, 395 (1993).
[3] G. A. Baker, B. G. Nickel, and D. I. Meiron, Phys. Rev. B **17**, 1365 (1978).
[4] J. C. Le Guillou and J. Zinn-Justin, Phys. Rev. B **21**, 3976 (1980).
[5] J. Zinn-Justin, *Quantum Field Theory and Critical Phenomena* (Clarendon Press, Oxford, 1996).
[6] S. A. Antonenko and A. I. Sokolov, Phys. Rev. E **51**, 1894 (1995).
[7] A. I. Sokolov, Fiz. Tverd. Tela (Leningrad) **40**, 1284 (1998) [Phys. Solid State **40**, 1169 (1998)].
[8] C. Bagnuls and C. Bervillier, Phys. Rev. B **41**, 402 (1990).
[9] C. M. Bender, S. Boettcher, and L. Lipatov, Phys. Rev. Lett. **68**, 3674 (1992).
[10] C. M. Bender and S. Boettcher, Phys. Rev. D **48**, 4919 (1993).
[11] N. Tetradis and C. Wetterich, Nucl. Phys. B **422**, 541 (1994).
[12] M. M. Tsypin, Phys. Rev. Lett. **73**, 2015 (1994).
[13] C. M. Bender and S. Boettcher, Phys. Rev. D **51**, 1875 (1995).
[14] T. Reisz, Phys. Lett. B **360**, 77 (1995).
[15] A. I. Sokolov, Fiz. Tverd. Tela (Leningrad) **38**, 640 (1996) [Phys. Solid State **38**, 354 (1996)].
[16] S.-Y. Zinn, S.-N. Lai, and M. E. Fisher, Phys. Rev. E **54**, 1176 (1996).
[17] A. I. Sokolov, V. A. Ul'kov, and E. V. Orlov, Zh. Fiz. Dosl. **1**, 362 (1997); e-print cond-mat/9803352.
[18] A. I. Sokolov, E. V. Orlov, and V. A. Ul'kov, Phys. Lett. A **227**, 255 (1997).
[19] R. Guida and J. Zinn-Justin, Nucl. Phys. B **489**, 626 (1997).
[20] T. R. Morris, Nucl. Phys. B **495**, 477 (1997).
[21] P. Butera and M. Comi, Phys. Rev. E **55**, 6391 (1997).
[22] M. M. Tsypin, Phys. Rev. B **55**, 8911 (1997).
[23] A. Pelissetto and E. Vicari, Nucl. Phys. B **522**, 605 (1998).
[24] A. I. Sokolov and E. V. Orlov, Phys. Rev. B **58**, 2395 (1998).
[25] I. O. Mayer, A. I. Sokolov, and B. N. Shalayev, Ferroelectrics **95**, 93 (1989); I. O. Mayer, J. Phys. A **22**, 2815 (1989).
[26] M. Campostrini, A. Pelissetto, P. Rossi, and E. Vicari, Nucl. Phys. B **459**, 207 (1996).
[27] A. Pelissetto and E. Vicari, Nucl. Phys. B **519**, 626 (1998).
[28] G. A. Baker, Jr., Phys. Rev. B **15**, 1552 (1977).
[29] G. A. Baker and N. Kawashima, Phys. Rev. Lett. **75**, 994 (1995).
[30] G. A. Baker, Jr. and P. Graves-Morris, *Padé Approximants* (Addison-Wesley, Reading, MA, 1981).
[31] R. Guida and J. Zinn-Justin, J. Phys. A **31**, 8103 (1998).